The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 4, 2018

Course: EE 313 Evans

Name: ____
Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	18		Sampling Sinusoids
2	18		Harmonics
3	24		Fourier Series Analysis
4	24		Undersampling
5	16		Potpourri
Total	100		

Problem 1.1 *Sampling Sinusoids*. 18 points.

Consider the sinusoidal signal $x(t) = \cos(2\,\pi\,f_0\,t + \theta)$ for continuous-time frequency f_0 in Hz.

We then sample $x(t)$ at a sampling rate f_s in Hz to produce a discrete-time signal $x[n]$.

(a) Derive the formula for $x[n]$ by sampling $x(t)$. *6 points.*

(b) Based on your answer in part (a), give a formula for the discrete-time frequency $\hat{\omega}_0$ in of $x[n]$ in terms of the continuous-time frequency f_0 and sampling rate f_s. Units of $\hat{\omega}_0$ are in rad/sample. *6 points*.

(c) For continuous-time frequency $f_0 = 440$ Hz and sampling rate $f_s = 8000$ Hz,

i. What is the smallest discrete-time period for $x[n]$? Why? *3 points.*

ii. How many continuous-time periods of $x(t)$ are in the smallest discrete-time period of $x[n]$? Why? *3 points.*

Problem 1.2 *Harmonics.* 18 points.

(a) ***Virtual Bass.*** Human auditory systems have the ability to perceive a frequency when the frequency is not present but many of its harmonics are present. Consider a less capable audio speaker, which can only play frequencies between 200 Hz and 1000 Hz. The speaker plays an audio clip and produces principal frequencies of 210 Hz, 315 Hz, 420 Hz, 525 Hz, 630 Hz, 735 Hz, 840 Hz, and 945 Hz *at the same time*. What additional principal frequency could a human listener perceive? *6 points*.

(b) ***Nonlinearity Effect.*** Consider the signal $x(t) = \cos^3(2\,\pi\, f_0\, t)$. Write the signal using the Fourier series synthesis formula

$$x(t) = \sum_{k=-N}^{N} a_k e^{j2\pi(k f_0)t}$$

i. What is the value of N? *3 points*.

ii. Give the values of all of the Fourier series coefficients a_k for $k = -N, \ldots, 0, \ldots, N$. *9 points*.

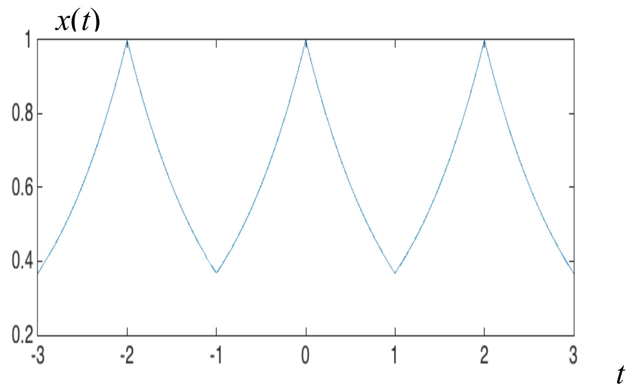
Problem 1.3 *Fourier Series Analysis.* 24 points.

A periodic signal $x(t)$ is defined over its fundamental period of duration T_0 as

$$e^{t} \quad \text{for } -\frac{T_0}{2} \leq t < 0$$

$$e^{-t} \quad \text{for } 0 \leq t < \frac{T_0}{2}$$

and plotted over three fundamental periods on the right.



Compute the Fourier series coefficients using

$$a_k = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-j2\pi(k f_0)t} dt$$

by answering the questions below.

(a) What is the value of T_0? How did you compute it? *6 points*.

(b) What is the value of a_0? What does the value of a_0 represent in $x(t)$? *6 points.*

(c) Give a formula for a_k. *12 points*.

Problem 1.4. *Undersampling.* 24 points**.**

In certain systems, the main alternating power frequency f_0 in Hz and its odd harmonics ($3f_0$, $5f_0$, etc.) cause interference. This problem will explore the effect of undersampling on the interference.

For simplicity, model the interference signal $g(t)$ as having the first and third harmonics of f_0:

$$g(t) = \cos(2\,\pi\,f_0\,t) + \cos(2\,\pi\,(3\,f_0)\,t)$$

Let $f_0 = 60$ Hz as it is in the US. (In many other countries, $f_0 = 50$ Hz.)

(a) Draw the spectrum of $g(t)$. Include positive and negative continuous-time frequencies. *6 points*.

(b) When sampling $g(t)$ at a sampling rate $f_s = 2f_0 = 120$ Hz obtain a formula for $g[n]$. *3 points*.

(c) Draw the spectrum of $g[n]$ in part (b) for discrete-time frequencies in the interval $[-\pi, \pi]$. *6 points*.

(d) When sampling $g(t)$ at a sampling rate $f_s = 4f_0 = 240$ Hz obtain a formula for $g[n]$. *3 points*.

(e) Draw the spectrum of $g[n]$ in part (d) for discrete-time frequencies in the interval $[-\pi, \pi]$. *6 points*.

Problem 1.5. *Potpourri.* 16 points.

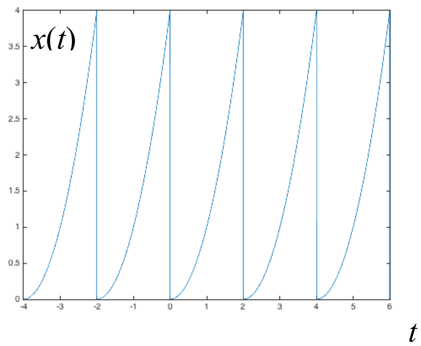
(a) Consider the periodic signal shown on the right with a fundamental period of T_0 = 2 seconds. Over one fundamental period,

$$x(t) = t^2$$

If we keep a large but finite number of Fourier series coefficients, explain whether or not the Fourier synthesis will suffer from Gibbs phenomenon.

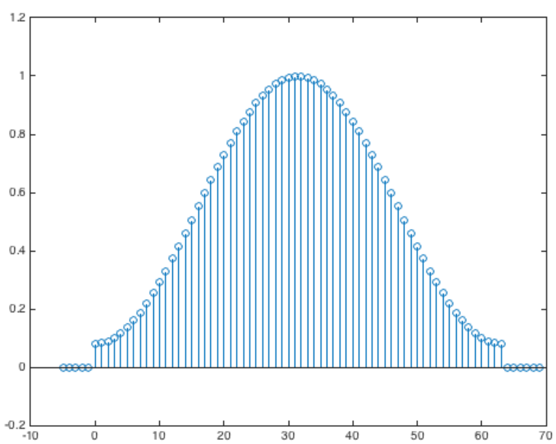
In your answer, please explain what Gibbs phenomenon is.

8 points.

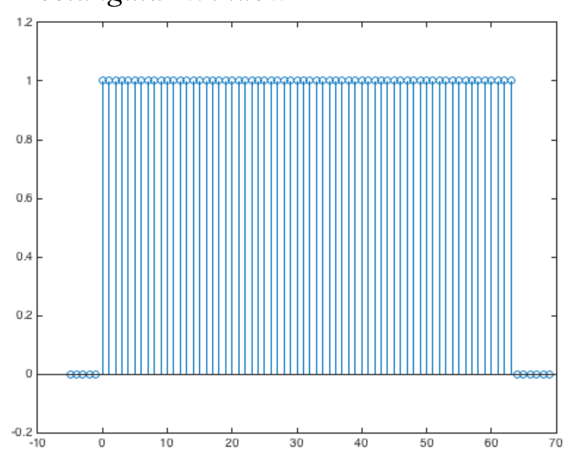


(b) In spectrogram calculations, explain why it is generally advantageous to use a Hamming window instead of a rectangular window to weight the amplitudes in each segment (block) of samples. Stem plots of Hamming and rectangular windows of length 64 samples are given below. *8 points*.

Hamming Window



Rectangular Window



MATLAB Code

```
% 1.3
T0 = 2;
f0 = 2;
fs = 1000*f0;
Ts = 1/fs;
Np = 3;
t = -Np*T0/2 : Ts : Np*T0/2;
tmod = mod(t - T0/2, T0) - T0/2;
x = exp( -abs(tmod) );
plot(t, x);
% Manually changed the font size to 20pt
% using the plot manager window

% 1.5(a) Periodic signal
T0 = 2;
f0 = 1/2;
fs = 2000*f0;
Ts = 1 / fs;
t = -2*T0 : Ts : 3*T0;
x = mod(t, T0) .^ 2;
plot(t, x);

% 1.5(b) Windows
n = -5 : 69;
v = zeros(1, 75);
v(6:69) = hamming(64);
figure;
stem(n, v);
ylim( [-0.2 1.2] );

n = -5 : 69;
v = zeros(1, 75);
v(6:69) = ones(1, 64);
figure;
stem(n, v);
ylim( [-0.2 1.2] );
```

References

Problem 1.2(a)

The principal frequencies correspond to the English phoneme ‘aw’ as reported on page 5 of http://www.physics.indiana.edu/~courses/p109/P109fa08/11.pdf

The description of the “missing frequency” effect comes from https://en.wikipedia.org/wiki/Missing_fundamental